Creating Confusion

Supplementary Online Appendix

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This appendix is organized as follows. In Appendix D we provide proofs of additional results omitted from the main text. In Appendix E we provide further details on the knife-edge case $c = 1$. In Appendix F we show that the expositional device of assuming that the coefficients in the receivers' strategy sum to one is without loss of generality.

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D Omitted proofs

In this appendix we provide proofs of results omitted from the main text. We first state and prove two supplementary lemmas used in the proof of Proposition 5 in the main text. We then provide proofs of Remark 1 and Remark 2 from the main text.

D.1 Supplementary lemmas

Supplementary Lemma 1. The total derivative of the receivers' equilibrium loss l^* with respect to α_x is strictly positive if and only if

$$F(k^*) := k^{*4} - 2k^{*3} + 2ck^* - c^2 > 0 \tag{D1}$$

Proof. Recall that $l^* = l(\delta^*; \alpha_x)$ where

$$l(\delta; \alpha_x) = \frac{(1-\lambda)}{(1-\delta)^2(1-\lambda)\alpha_x + \alpha_z} \tag{D2}$$

From this we obtain

$$\frac{dl^*}{d\alpha_x} > 0 \qquad \Leftrightarrow \qquad (1-\delta^*) - 2\alpha_x \frac{d\delta^*}{d\alpha_x} < 0 \tag{D3}$$

Equivalently, if and only if

$$\frac{d\delta^*}{d\alpha_x} > \frac{1}{2\alpha_x}(1-\delta^*) > 0 \tag{D4}$$

Now recall that in equilibrium the sender's manipulation depends on α_x only via the receivers' response coefficient, $\delta^*(\alpha_x) = \delta(k^*(\alpha_x))$, so that

$$\frac{d\delta^*}{d\alpha_x} = \delta'(k^*)\frac{dk^*}{d\alpha_x} \tag{D5}$$

So we can write condition (D4) as

$$\delta'(k^*)\frac{dk^*}{d\alpha_x} > \frac{1}{2\alpha_x}(1-\delta^*) > 0 \tag{D6}$$

Applying the implicit function theorem to the equilibrium condition (A2) from the main text we have

$$\frac{dk^*}{d\alpha_x} = \frac{\frac{\alpha_z}{(1-\lambda)\alpha_x}\frac{k^*}{\alpha_x}}{\frac{\alpha_z}{(1-\lambda)\alpha_x} - R'(k^*)} > 0 \tag{D7}$$

where $R(k)$ is defined in (A2) in the main text. Plugging this into (D6) and simplifying we get the equivalent condition

$$\frac{\alpha_z}{(1-\lambda)\alpha_x}\left(\delta'(k^*)k^* - \frac{1}{2}(1-\delta^*)\right) > -\frac{1}{2}(1-\delta^*)R'(k^*) \tag{D8}$$

Now observe from (A7) that

$$\delta'(k)k - \frac{1}{2}(1-\delta) = \frac{1}{2}\left(\frac{1}{c-k^2}\right)^2\left(k^3 - 3ck^2 + 3ck - c^2\right) \tag{D9}$$

and that using the formula for $R'(k)$ given in (A3) above we can calculate that

$$\frac{1}{2}(1-\delta)R'(k) = \frac{1}{2}\left(\frac{1}{c-k^2}\right)^2 R(k)\frac{1}{1-k}P(k) \tag{D10}$$

where $P(k)$ is also defined in (A3) above. Plugging these calculations back into (D8) gives

$$\frac{\alpha_z}{(1-\lambda)\alpha_x}\left(\frac{1}{2}\left(\frac{1}{c-k^{*2}}\right)^2\left(k^{*3} - 3ck^{*2} + 3ck^* - c^2\right)\right) > -\frac{1}{2}\left(\frac{1}{c-k^{*2}}\right)^2 R(k^*)\frac{1}{1-k^*}P(k^*) \tag{D11}$$

Canceling common terms gives the condition

$$\frac{\alpha_z}{(1-\lambda)\alpha_x}\left(k^{*3}-3ck^{*2}+3ck^*-c^2\right) > -R(k^*)\frac{1}{1-k^*}P(k^*) \tag{D12}$$

Using the equilibrium condition $L(k^*) = R(k^*)$ from (A2) and $\alpha = (1-\lambda)\alpha_x/\alpha_z$ gives

$$\frac{\alpha_z}{(1-\lambda)\alpha_x}\left(k^{*3}-3ck^{*2}+3ck^*-c^2\right) > -\frac{\alpha_z}{(1-\lambda)\alpha_x}\frac{k^*}{1-k^*}P(k^*) \tag{D13}$$

Using the definition of $P(k)$ and canceling more common terms gives the condition

$$k^{*4}-2k^{*3}+2ck^*-c^2 > 0 \tag{D14}$$

□

Supplementary Lemma 2. Define

$$F(k^*) := k^{*4}-2k^{*3}+2ck^*-c^2 \tag{D15}$$

(i) If $c > 1$, then $F(k^*) < 0$;

(ii) If $c < 1$, there is an interval $(\underline{k}, \overline{k})$ with $0 < \underline{k} < \overline{k} < 1$ such that $F(k) > 0$ for $k \in (\underline{k}, \overline{k})$ and $F(k) \leq 0$ otherwise. Moreover, the cutoffs are on either side of c so that $0 < \underline{k} < c < \overline{k} < 1$.

Proof. Write $F(k) = J(k;c) - G(k)$ where $J(k;c) := 2ck - c^2$ and $G(k) := 2k^3 - k^4$. Observe that $G(0) = 0$, $G(1) = 1$, $G(k) < k$ for all k; $G'(k) = 2k^2(3-2k) \geq 0$ with $G'(0) = 0$ and $G'(1) = 2$; and $G''(k) = 12k(1-k) \geq 0$ so that $G'(k) \leq G'(1) = 2$ for all k. Further observe that $J(0;c) = -c^2 < 0$, $J(1;c) = 2c - c^2 \leq 1$ (with equality if $c = 1$) and $J'(k;c) = 2c > 0$ for all k so that $J(k;c) \leq J(1;c) = 2c - c^2 \leq 1$ for all k, c. These imply $F(0) = J(0;c) - G(0) = -c^2 < 0$ and $F(1) = J(1;c) - G(1) = 2c - c^2 - 1 \leq 0$ (with equality if $c = 1$); $F'(k) = J'(k;c) - G'(k) = 2c - G'(k)$ and $F''(k) = -G''(k) \leq 0$. Since $G'(k) \leq 2$ we have

$$F'(k) = J'(k;c) - G'(k) = 2c - G'(k) \geq 2c - 2 = 2(c-1) \tag{D16}$$

For part (i) $c > 1$. Then $F'(k) \geq 2(c-1) > 0$ so $F(k)$ is strictly increasing from $F(0) = -c^2 < 0$ to $F(1) = 2c - c^2 - 1 < 0$ so that $F(k) < 0$ for all k.

For part (ii) $c < 1$. Then since $G'(k)$ is monotone increasing from $G'(0) = 0$ to $G'(1) = 2$ there is a unique critical point $\tilde{k}$ such that

$$F'(\tilde{k}) = 0 \qquad \Leftrightarrow \qquad 2c = G'(\tilde{k}) \tag{D17}$$

Since $F''(k) \leq 0$, this critical point maximizes $F(k)$ hence

$$F(k) \leq \max_{k\in[0,1]} F(k) = F(\tilde{k}) \tag{D18}$$

and observe that if we take $k = c < 1$ (which is feasible since here $c < 1$) then we have

$$F(c) = J(c;c) - G(c) = 2c^2 - c^2 - G(c) = c^2 - 2c^3 + c^4 = c^2(1 - 2c + c^2) > 0 \tag{D19}$$

so that indeed

$$F(\tilde{k}) \geq F(c) > 0 \tag{D20}$$

Hence for $c < 1$ there exist k such that $F(k) > 0$. More precisely, the function $F(k)$ increases from $F(0) = -c^2 < 0$ to a lower cutoff $\underline{k} \in (0, \tilde{k})$ defined by $F(\underline{k}) = 0$. The function $F(k)$ keeps increasing until it reaches the critical point $\tilde{k}$ at which $F'(\tilde{k}) = 0$ and $F(\tilde{k}) > 0$. From there $F(k)$ decreases, crossing zero again at a higher cutoff $\overline{k} \in (\tilde{k}, 1)$ defined by $F(\overline{k}) = 0$ and keeps decreasing until $F(1) = 2c - c^2 - 1 < 0$ (since $c < 1$).

So for $c < 1$ there is an interval $(\underline{k}, \overline{k})$ with $0 < \underline{k} < \overline{k} < 1$ such that $F(k) > 0$ for $k \in (\underline{k}, \overline{k})$ and $F(k) \leq 0$ otherwise. For $c < 1$ these critical points are defined by the roots of $F(k;c) = 0$. Observe that since $F(c) > 0$ yet $\underline{k}$ is the first k for which $F(k) = 0$ it must be the case that $\underline{k} < c$. Likewise since $F(\overline{k}) = 0$ it must also be the case that $\overline{k} > c$. In short, the cutoffs are on either side of c so that $0 < \underline{k} < c < \overline{k} < 1$. □

D.2 Additional remarks

Proof of Remark 1.

From Proposition 1 in the main text, we know that $k^* < \min(c, 1)$. As $\alpha \to \infty$, $L(k)$ in (A2) approaches zero. To make the equilibrium condition hold, $R(k)$ in (A2) needs to approach zero too. If $c < 1$, this condition is achieved by $k \to c$. If $c > 1$, this condition is achieved by $k \to 1$. The limiting results of δ^* are implied by Lemma 2 in the main text when $k^* \to \min(c, 1)$. $\square$

Proof of Remark 2.

Suppose that $\lambda < 0$ and $c < c^*_{nm}(\alpha)$ so that $k^* < k^*_{nm}$. We can rewrite the condition (C6) for the sender's manipulation to backfire as

$$\frac{k^{*2}}{c - k^*} \frac{(1 - k^*)^2}{k^*_{nm} - k} < 1 - \lambda(k^*_{nm} + k^*). \tag{D21}$$

Using the equilibrium condition (A2) and the sender's best response (22), we have:

$$\frac{k^{*2}}{c - k^*} = \alpha \frac{c(1 - k^*)k^*}{(c - k^{*2})^2} = \alpha\delta^* \left(1 + \frac{k^*}{1 - k^*}\delta^*\right). \tag{D22}$$

As $c \to 0$, the RHS of (D21) converges to

$$\lim_{c \to 0} \left(1 - \lambda(k^*_{nm} + k^*)\right) = 1 - \lambda \frac{\alpha}{\alpha + 1} \tag{D23}$$

since $k^* \to 0$ as $c \to 0$. Recall that as $c \to 0$, $\delta^* \to 1$, the LHS of (D21) converges to

$$\lim_{c \to 0} \left(\frac{k^{*2}}{c - k^*} \frac{(1 - k^*)^2}{k^*_{nm} - k} \right) = \alpha \frac{1}{\frac{\alpha}{\alpha+1}} = \alpha + 1. \tag{D24}$$

Therefore, the condition (D21) must hold when c is small enough if

$$\alpha + 1 < 1 - \lambda \frac{\alpha}{\alpha + 1}. \tag{D25}$$

Since $\alpha = (1 - \lambda)\alpha_x/\alpha_z$, the inequality above can be rewritten as

$$\alpha_x + \alpha_z < (\alpha_x - \alpha_z)\lambda. \tag{D26}$$

Given that $\lambda < 0$, the necessary condition for the inequality above to hold is $\alpha_x < \alpha_z$. When it is the case, the inequality above is equivalent to

$$\lambda < -\frac{\alpha_x + \alpha_z}{\alpha_z - \alpha_x} < -1. \tag{D27}$$

In sum, when $\alpha_x < \alpha_z$, for each λ satisfying the inequality (D27), there must exist a cutoff $\underline{c}^*$ such that for all $c < \underline{c}^*$, the condition (D21) for the sender's manipulation to backfire holds. Finally, the cutoff $\underline{c}^*$ must be lower than $c^*_{nm}(\alpha)$ so that $c < \underline{c}^*$ is sufficient for $k^* < k^*_{nm}$. $\square$

E Knife-edge case $c = 1$

In this appendix we provide more details on the knife-edge case where the costs of manipulation are $c = 1$ exactly. We first explain why this case needs to be handled separately (when the relative precision α is sufficiently high) and explain how the equilibrium is determined in this case. We then explain the comparative statics of the equilibrium local to $c = 1$ and how this can cause the equilibrium amount of manipulation δ^* to jump discretely at $c = 1$. We then give some intuition for large changes in δ^* near $c = 1$ and how to interpret this specific parameter value.

Preliminaries. There is no issue with $c = 1$ if the relative precision $\alpha \leq 4$. The issues with $c = 1$ arise only if $\alpha > 4$. To see this, first recall from Lemma 1 that if $\alpha > 1$ the receivers' best response $k(\delta; \alpha)$ is increasing in δ on the interval $[0, \hat{\delta}(\alpha)]$ and obtains its maximum at $\delta = \hat{\delta}(\alpha) = 1 - 1/\sqrt{\alpha} \in (0, 1)$. At the maximum, the receivers' best response takes on the value $k(\hat{\delta}(\alpha); \alpha) = \sqrt{\alpha}/2$. Hence for $\alpha > 4$ the maximum value exceeds 1. Moreover, by continuity of the best response in δ if $\alpha > 4$ there is an interval of δ such that $k(\delta; \alpha) > 1$. The boundaries of this interval $(\underline{\delta}(\alpha), \overline{\delta}(\alpha))$ are given by the roots of $k(\delta; \alpha) = 1$, which work out to be

$$\underline{\delta}(\alpha)\,,\, \overline{\delta}(\alpha) = \frac{1}{2}\left(1 \pm \sqrt{1 - (4/\alpha)}\right), \qquad \alpha \geq 4 \tag{E1}$$

Observe that this interval is symmetric and centred on $1/2$ with a width of

$$\overline{\delta}(\alpha) - \underline{\delta}(\alpha) = \sqrt{1 - (4/\alpha)} \geq 0, \qquad \alpha \geq 4 \tag{E2}$$

If $\alpha = 4$, we have $\underline{\delta}(4) = \overline{\delta}(4) = 1/2$ but as α increases the width of the interval $(\underline{\delta}(\alpha), \overline{\delta}(\alpha))$ expands around $1/2$ with the boundaries $\underline{\delta}(\alpha) \to 0^+$ and $\overline{\delta}(\alpha) \to 1^-$ as $\alpha \to \infty$. Now recall from Proposition 1 that only $k \in [0, \min(c, 1)]$ and $\delta \in [0, 1]$ are candidates for an equilibrium. So if $\alpha > 4$ then none of the values of $\delta \in (\underline{\delta}(\alpha), \overline{\delta}(\alpha))$ are candidates for an equilibrium.

General case, $c \neq 1$. Now consider the sender's best response $\delta(k; c)$ parameterized by $c \neq 1$ and suppose $\alpha > 4$. When $c \neq 1$, the sender's objective always depends on δ over the entire support $k \in [0, \min(c, 1)]$. As proved in Proposition 1, there is a unique intersection between the sender's and the receivers' best responses. As illustrated below, if $c < 1$ the sender's best response $\delta(k; c)$ must lie above $\delta(k; 1) = k/(1 + k)$ and hence the equilibrium point k^*, δ^* must be on the "upper branch" of $k(\delta; \alpha)$ where $\delta^* > \overline{\delta}(\alpha)$. But for the same value of α and instead $c > 1$ the equilibrium point k^*, δ^* must be on the "lower branch" of $k(\delta; \alpha)$ where $\delta^* < \underline{\delta}(\alpha)$ because the sender's best response $\delta(k; c)$ lies below $\delta(k; 1) = k/(1 + k)$.

Knife-edge case, $c = 1$. Recall from (38) that the sender's objective can be written

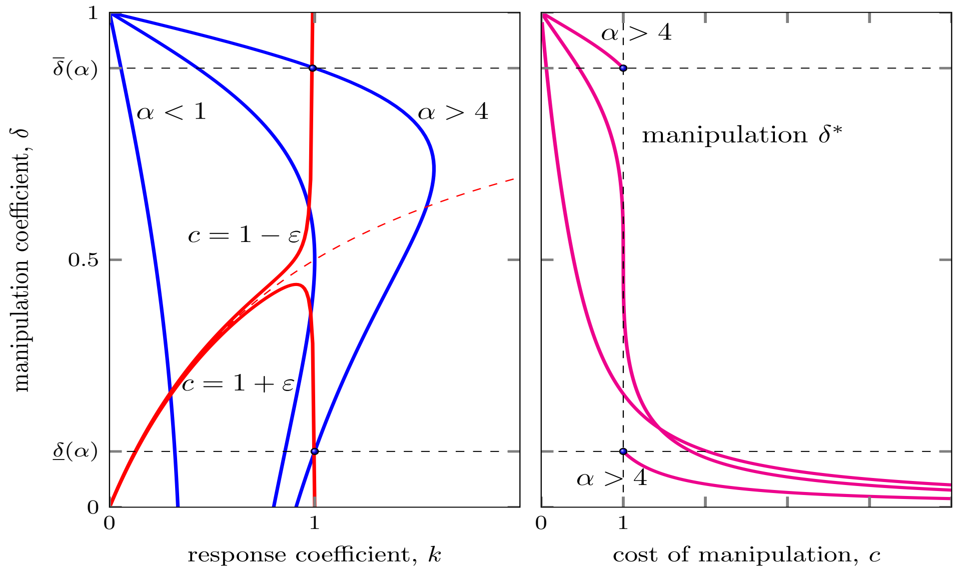
$$V(\delta, k) = \frac{1}{\alpha_z}\left(B(\delta, k) - C(\delta)\right) + \frac{1}{\alpha_x}k^2 \tag{E3}$$

with benefit $B(\delta, k) = (k\delta + 1 - k)^2$ and cost of manipulation $C(\delta) = c\delta^2$. When $c = 1$ exactly the relevant part of the sender's objective becomes

$$B(\delta, k) - C(\delta) = (k\delta + 1 - k)^2 - \delta^2 \tag{E4}$$

When $k \neq 1$, the sender's best response is $\delta(k; 1) = k/(1 + k)$, which is increasing in k and approaches $1/2$ as $k \to 1$. But when $k = 1$, the sender's objective is independent of δ and in turn the sender is indifferent to δ. Any $\delta \in [0, 1]$ is a best response to $k = 1$. In this case, the equilibrium is entirely determined by the receivers' best response. If $\alpha < 4$, the receivers' best response $k(\delta; \alpha) < 1$ so that $k^* = 1$ is never an equilibrium. If $\alpha = 4$ exactly there is a unique equilibrium with $k^* = 1$ and $\delta^* = 1/2$. If $\alpha > 4$ then there are two equilibria corresponding to the two roots of $k(\delta; \alpha) = 1$, namely (i) $k^* = 1$ and $\delta^* = \underline{\delta}(\alpha)$ and (ii) $k^* = 1$ and $\delta^* = \overline{\delta}(\alpha)$. That is, there is a "low manipulation" equilibrium and a "high manipulation" equilibrium and in both of these equilibria the receivers are maximally responsive to their signals.

Importantly, this equilibrium configuration is negligible in the sense that for any c arbitrarily close to 1 there is a unique (linear) equilibrium for any $\alpha > 0$.



Discontinuity at $c = 1$ and jump in the amount of manipulation δ^*

The left panel shows the receivers' best response $k(\delta;\alpha)$ for $\alpha < 1$, $\alpha = 4$ and $\alpha > 4$ (blue) and the sender's best response $\delta(k;c)$ for $c = 1-\varepsilon$, $c = 1$, and $c = 1+\varepsilon$ (red). For $\alpha > 4$, in the limit as $c \to 1^-$ the equilibrium is at $k^* = 1, \delta^* = \overline{\delta}(\alpha)$ but in the limit as $c \to 1^+$ the equilibrium is at $k^* = 1, \delta^* = \underline{\delta}(\alpha)$. For $\alpha > 4$ and $c = 1$ both of these are equilibria because for this knife-edge special case the sender is indifferent between $\underline{\delta}(\alpha)$ and $\overline{\delta}(\alpha)$. The right panel shows the equilibrium manipulation δ^* as a function of c for $\alpha < 1$, $\alpha = 4$ and $\alpha > 4$. For $\alpha \leq 4$, the manipulation δ^* is continuous in c. But for $\alpha > 4$ the manipulation jumps discontinuously at $c = 1$. In the limit as $\alpha \to \infty$ the boundaries $\underline{\delta}(\alpha) \to 0^+$ and $\overline{\delta}(\alpha) \to 1^+$ so that the manipulation jumps by the maximum possible amount, from $\delta^* = 0$ if $c < 1$ to $\delta^* = 1$ if $c > 1$.

Intuition for large changes in manipulation near $c = 1$. Now consider the sensitivity of the equilibrium amount of manipulation to changes in c near $c = 1$. The main case of interest is where $\alpha \to \infty$ so that there is no fundamental uncertainty and, absent manipulation, the receivers would be very responsive to their signals. In this case, $k \to \min(c, 1)$. First, suppose that $c > 1$ so that $k \to 1$. Then the relevant part of the sender's objective from (E3) simplifies to

$$B(\delta, 1) - C(\delta) = (1-c)\delta^2 \tag{E5}$$

so that for any $c > 1$ the sender will choose $\delta = 0$. Next, suppose instead that $c < 1$ so that $k \to c$. In this case the relevant part of the sender's objective simplifies to

$$B(\delta, c) - C(\delta) = -c(1-c)\delta^2 + 2c(1-c)\delta + (1-c)^2 \tag{E6}$$

so that for any $c < 1$ the sender will choose $\delta = 1$. In short, as $\alpha \to \infty$, the sender's manipulation is a *step function* in c, with $\delta = 1$ for all $c < 1$ and $\delta = 0$ for all $c > 1$.

What is the meaning of $c = 1$? So given that the amount of manipulation can be extremely sensitive to c near $c = 1$, what does $c = 1$ mean? Recall that in the sender's objective (5) the gross benefit $\int_0^1 (a_i - \theta)^2\, di$ has a coefficient normalized to 1. If instead we had written the objective with $b\int_0^1 (a_i - \theta)^2\, di$ for some $b > 0$ then throughout the analysis the relevant parameter would be the cost/benefit ratio c/b and the critical point would be where the cost/benefit ratio is $c/b = 1$. In this parameterization, the sender's equilibrium manipulation is extremely sensitive to changes in either c or b in the vicinity of $c/b = 1$. With α high and costs and benefits *evenly poised*, a small decrease in b or small increase in c would lead to a large reduction in manipulation.

F Coefficients sum to one

In this appendix we show that writing the receivers' linear strategy as $a(x_i, z) = kx_i + (1-k)z$ is without loss of generality. We show this with the more general version of our model that allows for strategic interactions among receivers, i.e., with the receivers' quadratic loss defined in (34).

Suppose that the receivers' linear strategy is

$$a_i = \beta_0 + \beta_1 x_i + \beta_2 z$$

for some coefficients $\beta_0, \beta_1, \beta_2$. We will show that in any linear equilibrium $\beta_0 = 0$ and $\beta_1 + \beta_2 = 1$. With this strategy, the aggregate A is

$$A = \beta_0 + \beta_1 y + \beta_2 z$$

The sender's problem is then to choose y to maximize

$$\int_0^1 (a_i - \theta)^2 \, di - c(y-\theta)^2 = (\beta_0 + \beta_1 x_i + \beta_2 z - \theta)^2 + \frac{1}{\alpha_x}\beta_1^2 - c(y-\theta)^2$$

The solution to this problem is

$$y = \gamma_0 + \gamma_1 \theta + \gamma_2 z$$

where

$$\gamma_0 = \frac{\beta_0 \beta_1}{c - \beta_1^2} \tag{F1}$$

$$\gamma_1 = \frac{c - \beta_1}{c - \beta_1^2} \tag{F2}$$

$$\gamma_2 = \frac{\beta_1 \beta_2}{c - \beta_1^2} \tag{F3}$$

But if the sender has the strategy $y = \gamma_0 + \gamma_1 \theta + \gamma_2 z$, the receivers' posterior expectation of θ is

$$\begin{aligned}\mathbb{E}[\theta \,|\, x_i, z] &= \frac{\gamma_1 \alpha_x}{\gamma_1^2 \alpha_x + \alpha_z}\left(\frac{1}{\gamma_1}(x_i - \gamma_2 z) - \frac{\gamma_0}{\gamma_1}\right) + \frac{\alpha_z}{\gamma_1^2 \alpha_x + \alpha_z} z \\ &= \frac{\gamma_1 \alpha_x}{\gamma_1^2 \alpha_x + \alpha_z} x_i + \frac{\alpha_z - \gamma_1 \alpha_x \gamma_2}{\gamma_1^2 \alpha_x + \alpha_z} z - \frac{\gamma_1 \alpha_x}{\gamma_1^2 \alpha_x + \alpha_z}\gamma_0\end{aligned}$$

And the equilibrium strategy of an individual receiver then satisfies

$$\begin{aligned}a_i &= \lambda \mathbb{E}[A \,|\, x_i, z] + (1-\lambda)\mathbb{E}[\theta \,|\, x_i, z] \\ &= \lambda \beta_1 \mathbb{E}[y \,|\, x_i, z] + (1-\lambda)\mathbb{E}[\theta \,|\, x_i, z] + \lambda \beta_2 z + \lambda \beta_0 \\ &= (\lambda \beta_1 \gamma_1 + (1-\lambda))\mathbb{E}[\theta \,|\, x_i, z] + \lambda(\beta_1 \gamma_2 + \beta_2) z + \lambda(\beta_1 \gamma_0 + \beta_0)\end{aligned}$$

Matching coefficients with $a_i = \beta_0 + \beta_1 x_i + \beta_2 z$ we then have

$$\beta_0 = -(\lambda \beta_1 \gamma_1 + (1-\lambda))\frac{\gamma_1 \alpha_x}{\gamma_1^2 \alpha_x + \alpha_z}\gamma_0 + \lambda(\beta_1 \gamma_0 + \beta_0) \tag{F4}$$

$$\beta_1 = \quad (\lambda \beta_1 \gamma_1 + (1-\lambda))\frac{\gamma_1 \alpha_x}{\gamma_1^2 \alpha_x + \alpha_z} \tag{F5}$$

$$\beta_2 = \quad (\lambda \beta_1 \gamma_1 + (1-\lambda))\frac{\alpha_z - \gamma_1 \alpha_x \gamma_2}{\gamma_1^2 \alpha_x + \alpha_z} + \lambda(\beta_1 \gamma_1 + \beta_2) \tag{F6}$$

Now observe that equations (F1) and (F4) together imply that the intercepts are $\beta_0 = \gamma_0 = 0$. Now observe from (F2)-(F3) and (F5)-(F6) that $\gamma_1 + \gamma_2 = 1$ implies $\beta_1 + \beta_2 = 1$ and vice-versa. So in one equilibrium the receivers' strategy takes the form $a_i = kx_i + (1-k)z$ where $k = \beta_1$ and the sender's strategy takes the form $y = (1-\delta)\theta + \delta z$ where $\delta = \gamma_2$. Hence from (F3) and (F5) we can write

$$\delta = \frac{k - k^2}{c - k^2}, \qquad k = \frac{(1-\delta)\alpha}{(1-\delta)^2 \alpha + 1}$$

where $\alpha := (1-\lambda)\alpha_x/\alpha_z$. These are the same as the best response formulas equations (13) and (18) in the main text and from Proposition 1 we know that there is a unique pair k^*, δ^* satisfying these conditions.